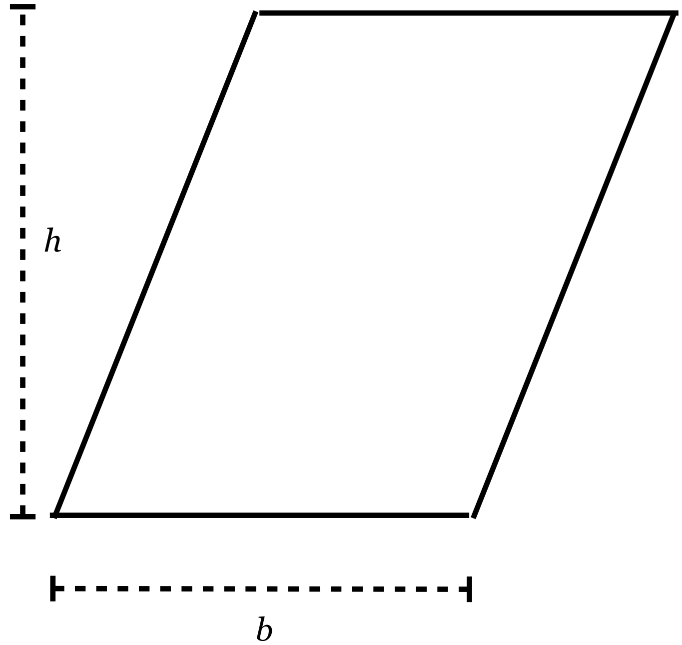
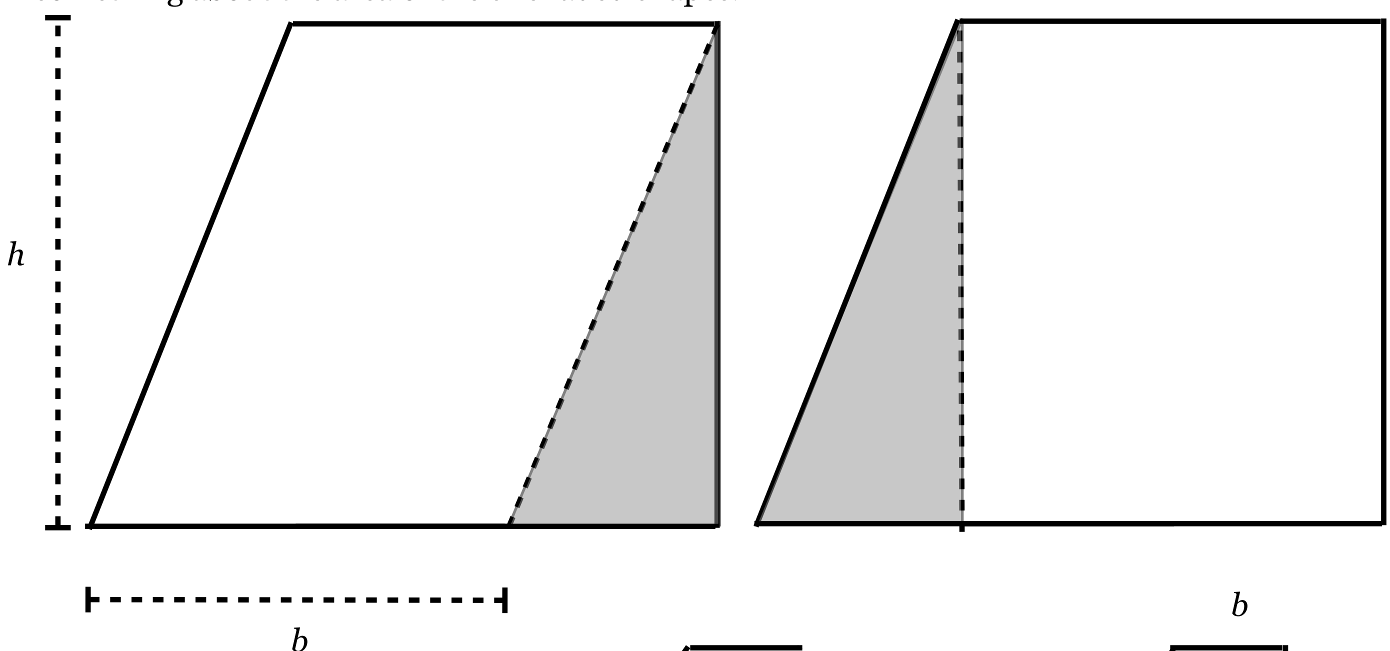
Cut Out Demonstrations of Area Formulas

For each of the following: (1) Use paper and scissors to make the construction. (2) Give a formula for the area of the shape and explain how it's justified by the construction. You may assume the formula for the area of a rectangle.

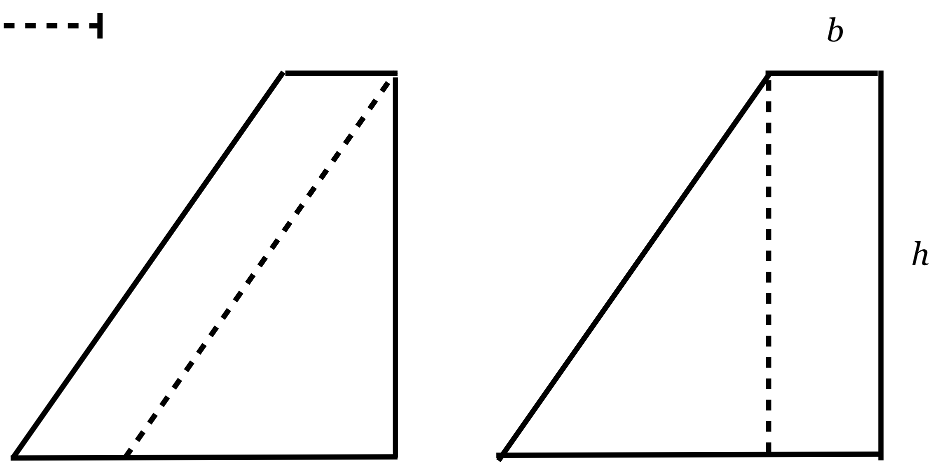
Parallelogram 1. Make one up-and-down cut, then rearrange the pieces to turn this into a b by h rectangle.



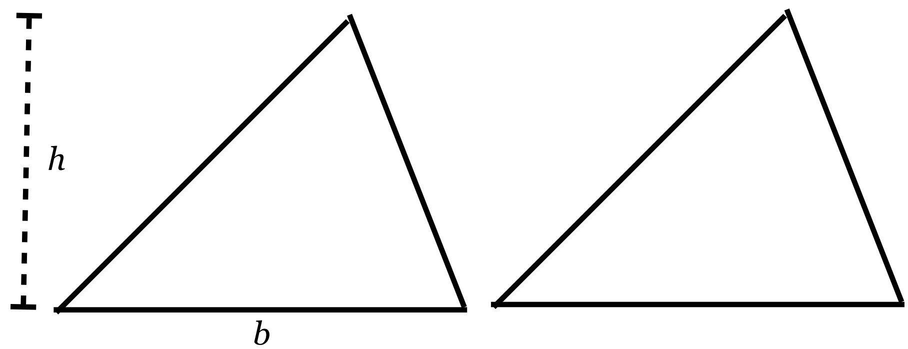
Parallelogram 2. The two trapezoids below are congruent. Cut along the two different dotted lines. Convince yourself that the two shaded triangles are congruent. Conclude something about the area of the unshaded shapes.



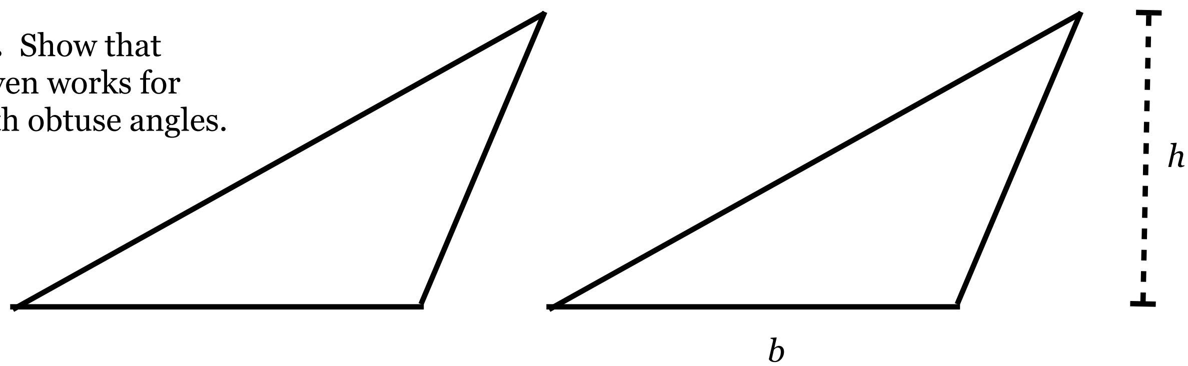
Parallelogram 3. Do Parallelogram 2, but for a very slanted parallelogram. Check that the same arguments hold.



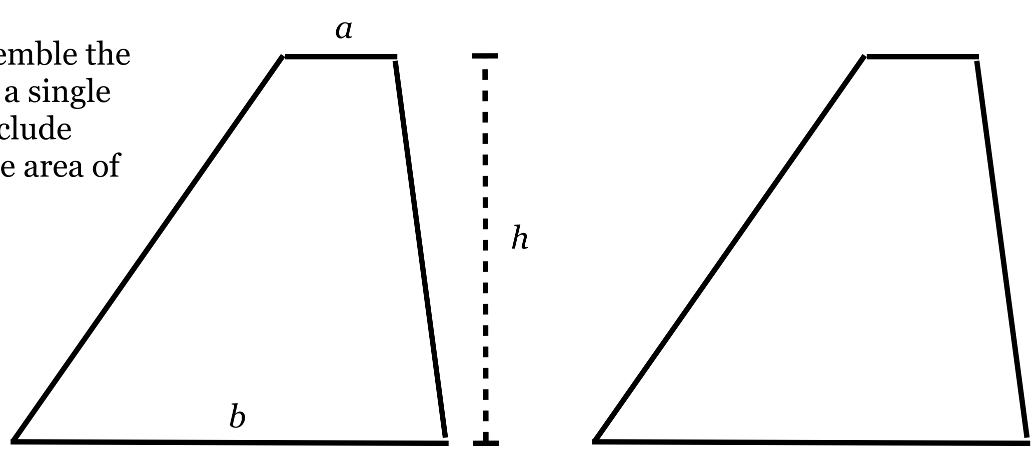
Triangle 1. Take two copies of a triangle and assemble them to make a parallelogram. Conclude something about the area of a single triangle.



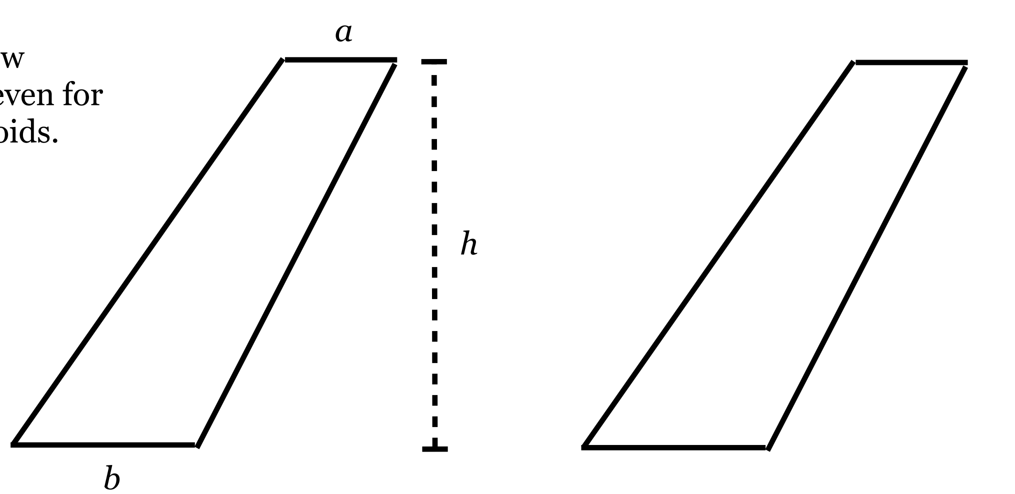
Triangle 2. Show that Triangle 1 even works for triangles with obtuse angles.



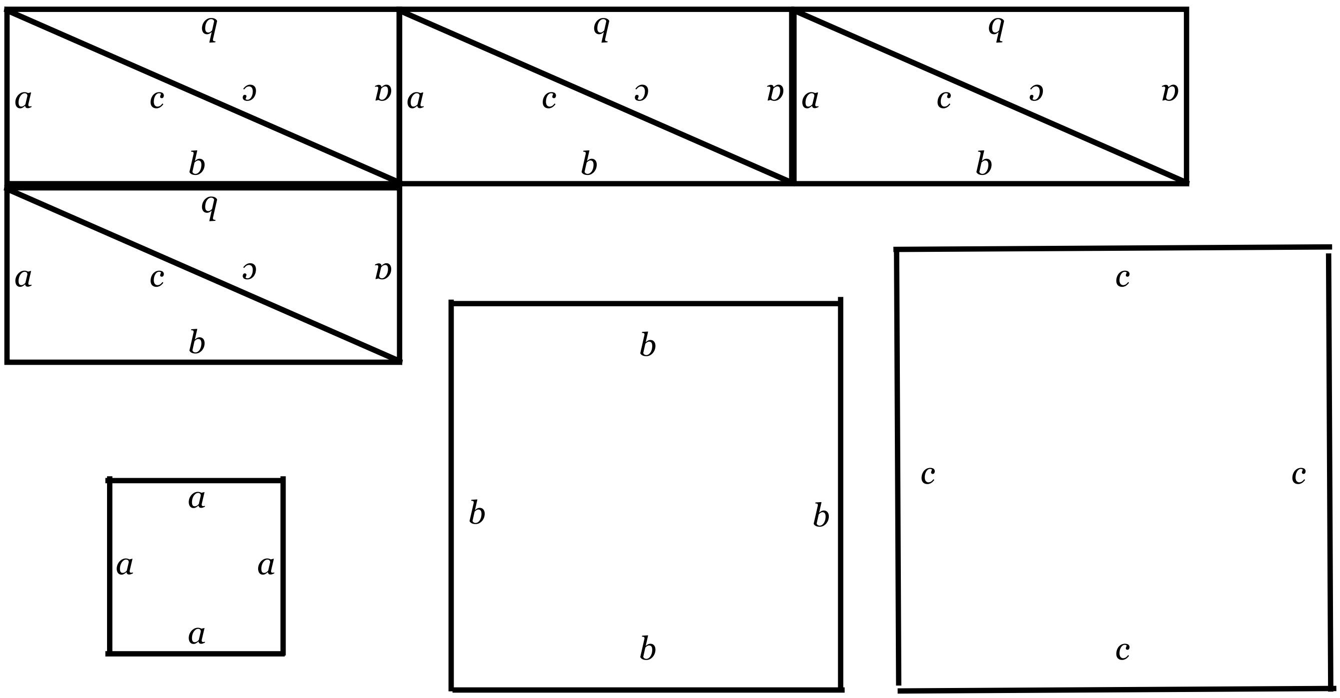
Trapezoid 1. Assemble the two trapezoids into a single parallelogram. Conclude something about the area of a single trapezoid.



Trapezoid 2. Show Trapezoid 1 works even for very slanted trapezoids.

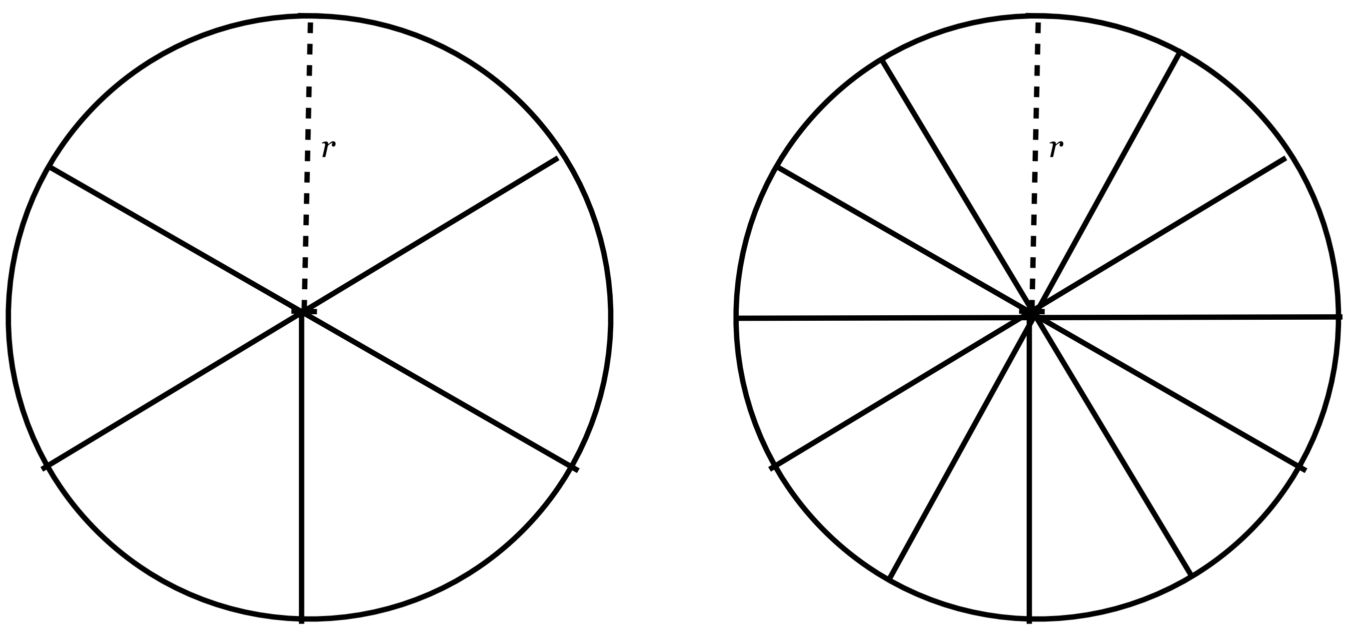


Puzzle: Make Two Equal Squares. Below are eight copies of a right triangle with legs *a, b* and *c*, and three squares of side *a, b* and *c*. Assemble these 11 pieces into two squares of equal area. Then use your figure to find a relationship between the areas of the squares. Then find a relationship between *a, b* and *c*.



Bonus Circle. Cut up the first circle and rearrange the shapes into a shape resembling a parallelogram. Write the approximate dimensions of this parallelogram in terms of *r* and calculate the area.

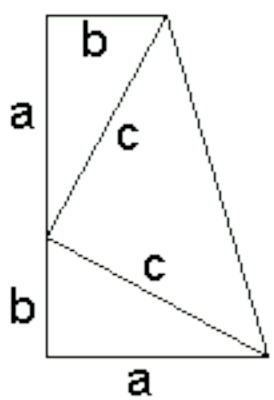
Rearrange the second circle similarly, and calculate the area of the parallelogram. Does the finer division result in something closer to a parallelogram?



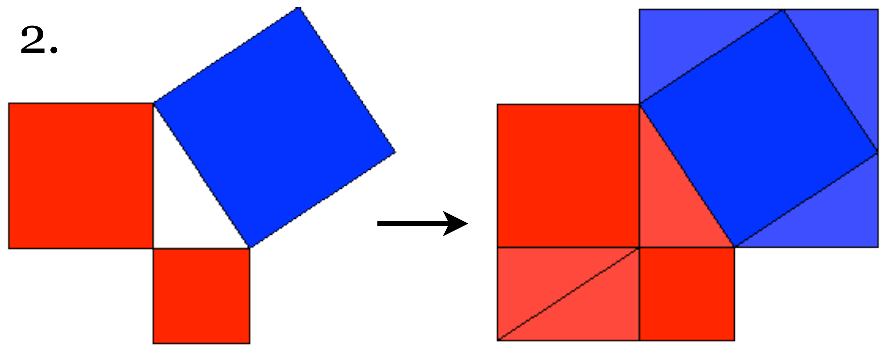
Wordless Pythagorean Theorem Proofs

Figure out how each of these diagrams proves the Pythagorean Theorem.
Hints at the bottom of the page.

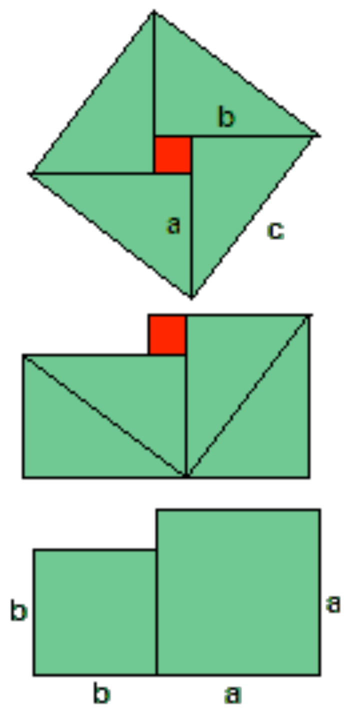
1.



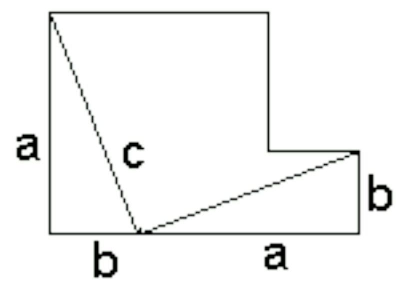
2.



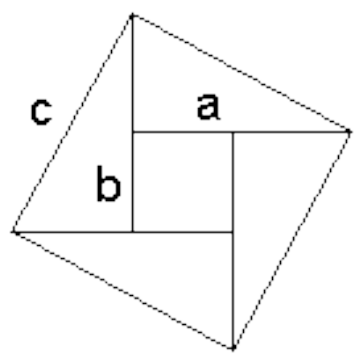
3.



4.



5.



Hints: (1) Don't those three triangles make a trapezoid? (2) What did I add to make the final shape? Isn't the final shape symmetric? (3) The second is a "hinge-swing" from the first. The third is a re-labeling of the second. (4) Move the left triangle to fill the L-notch to the top right. Move the right triangle to sit on top of the original shape. (5) What are the dimensions of the middle square in terms of *a* and *b*.